Question 1

(a)
(i) y, out (ii) x, in (iii) z, in (iv) x, out

(b)
Plane equation is \( Ax + By + Cz + D = 0 \)

where \( <A, B, C> \) is a normal vector to the plane.

Normal is found by taking cross product of two vectors formed from 3 points. Use \( \vec{AB} \) and \( \vec{AC} \):

\[
\vec{AB} = (7, 1, -2) - (3, 2, -12)
= <4, -1, 10 >
\]

\[
\vec{AC} = (6, 6, 5) - (3, 2, -12)
= <3, 4, 17 >
\]

\[
\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 4 & -1 & 10 \\ 3 & 4 & 17 \end{vmatrix}
\]

\[
= <-57, -38, 19 >
\]

\[
= <-3, -2, 1 >
\]

The plane equation is \( -3x - 2y + z + D = 0 \)

Use point \( A \) to find \( D \)

\[
-3x - 2y + z + D = 0
\]
\[
-3 \cdot 3 - 2 \cdot 2 + -12 + D = 0
\]
\[
D = 25
\]

The plane equation is

\[
-3x - 2y + z + 25 = 0
\]

(c)
Proceed as in .

Plane equation is

\( Ax + By + Cz + D = 0 \)

where \( <A, B, C> \) is a normal vector to the plane.

Normal is found by taking cross product of two vectors formed from 3 points. Use \( \vec{EF} \) and \( \vec{EG} \):
\( E(5,4,-1) \) \( F(7,0,5) \)

\[
\vec{EF} = (7,0,5) - (5,4,-1) \\
= <2,-4,6>
\]

\[
\vec{EG} = (9,2,5) - (5,4,-1) \\
= <4,-2,6>
\]

\[
\vec{EF} \times \vec{EG} = \begin{vmatrix} i & j & k \\ 2 & -4 & 6 \\ 4 & -2 & 6 \end{vmatrix} \\
= <-12,12,12> \\
= <-1,1,1>
\]

The plane equation is

\[-x + y + z + D = 0\]

Use point \( E(5,4,-1) \) to find \( D \)

\[
-x + y + z + D = 0 \\
-5 + 4 - 1 + D = 0 \\
D = 2
\]

The plane equation is

\[-x + y + z + 1 = 0\]

(d)

Rearranging the plane equation for the plane equation found in (b) and solving for \( z \) at \( x = 6 \) and \( y = 2 \) gives:

\[-3x - 2y + z + 25 = 0\]

\[
z = -25 + 3x + 2y \\
= -25 + 3 \cdot 6 + 2 \cdot 2 \\
= -3
\]

Rearranging the plane equation for the plane equation found in (c) and solving for \( z \) at \( x = 6 \) and \( y = 2 \) gives:

\[-x + y + z + 2 = 0\]

\[
z = -2 + x - y \\
= -2 + 6 - 2 \\
= 2
\]

Therefore the plane in (c) has the greater \( z \) value and will be seen.
(e)

Need to determine the drawing order. Do this by determining if one polygon lies wholly in front of the plane of the other. Can use either dot products or plane equations.

<table>
<thead>
<tr>
<th>Point</th>
<th>$x$</th>
<th>$y$</th>
<th>$z_{P1}$</th>
<th>$z_{P2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>2</td>
<td>-12</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>7</td>
<td>1</td>
<td>-2</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>-1</td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>6</td>
<td>-1</td>
<td>-3</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>4</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>7</td>
<td>0</td>
<td>-4</td>
<td>6</td>
</tr>
<tr>
<td>G</td>
<td>9</td>
<td>2</td>
<td>-3</td>
<td>3</td>
</tr>
</tbody>
</table>

From the table it is clear that P2 is in front of P1’s plane for points E, F and G, i.e., for all of P2. Therefore draw P2 first and P1 second.

(f)

A point $P$ lies to the left of a line segment $AB$ if $\vec{AB} \times \vec{AP}$ is positive. This follows from the definition of the cross product:

$$A \times B = |A||B| \sin \theta$$

For $0 \deg < \theta < 180 \deg$, $\sin \theta$ is positive. Therefore the cross product is positive (points out of page). For $180 \ deg < \theta < 360 \deg$, $\sin \theta$ is negative. Therefore the cross product is negative (points into page).

Use the cross product $\vec{AB} \times \vec{AP}$ to determine if $P$ lies to the “inside” (left) and another cross and the cross product $\vec{AB} \times \vec{AP}$ to determine if $P$ lies to the “inside” (left).

$$\vec{AB} = (2, 5) - (3, 9)$$
$$= <-1, -4>$$

$$\vec{AP} = (2.3, 7.2) - (3, 9)$$
$$= <-0.7, -1.8>$$

$$\vec{AB} \times \vec{AP} = \begin{vmatrix} -1 & -4 \\ -0.7 & -1.8 \end{vmatrix}$$
$$= 1.0$$

Therefore the point lies to the left.

**Question 2**

(b)

Steps:

1. Scale. Reflect about the x axis.

$$S_x = 1$$

$$S_y = -1$$

$$T_2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

3
2. Scale.

\[ S_x = \frac{\sqrt{2^2 + 4^2}}{4} = \sqrt{5}/2 \]

\[ S_y = \frac{\sqrt{2^2 + 1^2}}{1} = 1 \]

\[ T_1 = \begin{bmatrix} \sqrt{5}/2 \\ \sqrt{5} \\ 1 \end{bmatrix} \]

3. Rotate by \( \theta = tan^{-1}(2) \) deg

\[ T_3 = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \]

\[ T_3 = \begin{bmatrix} 1/\sqrt{5} & -2/\sqrt{5} \\ 2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix} \]

4. Translate to (4,3)

\[ T_4 = \begin{bmatrix} 1 & 4 \\ 1 & 3 \\ 1 \end{bmatrix} \]

The combined transformation matrix (CTM) is determined by multiplying the matrices together:

\[ CTM = T_1T_2T_3T_4 \]

\[ = \begin{bmatrix} 1 & 4 \\ 1 & 3 \\ 1 \end{bmatrix} \begin{bmatrix} \sqrt{5}/2 \\ \sqrt{5} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \]

\[ = \begin{bmatrix} 1 & 4 \\ 1 & 3 \\ 1 \end{bmatrix} \begin{bmatrix} 1/2 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} \]

\[ = \begin{bmatrix} 1/2 & 2 \\ 1 & -1 \end{bmatrix} \]

**Question 3**

(a)

Algorithm only.

ET stands for edge table. AET stands for active edge table.
1. set $y$ to smallest $y$ in ET
2. initialize AET as empty
3. repeat until AET and ET are both empty
   (a) move ET bucket to AET
   (b) sort AET on $x$
   (c) fill pixels on scan line with coordinate pairs
   (d) remove edges from AET where $y = y_{max}$
   (e) update each edge by adding its slope to $x$
   (f) increment $y$ by 1 to next scan line

(b)
(e)

\[
y = 1 \quad \text{pixel}(6,1)-\text{pixel}(10,1)
\]

\[
y = 3 \quad \text{pixel}(6,3)-\text{pixel}(11,3)
\]

\[
y = 4 \quad \text{pixel}(6,3)-\text{pixel}(12,3)
\]

Question 5

(a)

Use a hierarchical data structure — a directed acyclic graph (DAG) or a tree. In this case a binary tree is the simplest. Each node contains details of the material properties and geometry of each primitive, and the transformations which apply to it. There are two child pointers per node (to form a binary tree). One child pointer points to other nodes at the same level in the hierarchy. The other child pointer is to nodes deeper in the hierarchy.

```c
struct object {
    float tx, ty, tz; /* Translation relative to parent. */
    float rx, ry, rz; /* Rotation relative to parent. */
    float sx, sy, sz; /* Scaling relative to parent. */

    /* Other characteristics. For example, colour. */
```
object_type type; /* Type tag. */

/* Tree pointers. */
struct object *next_object; /* Link to object at same level. */
struct object *next_level; /* Link to object at next level. */
}

(b)

myWireBox(GLfloat width, GLfloat height, GLfloat depth)
{
    glPushMatrix();
    glScalef(width, height, depth);
    glutWireCube(1.0);
    glPopMatrix();
}

(c)

To render the model the data structure is essentially interpreted by traversing it and executing the commands which change graphics state or context and drawing the primitives. The essence of the structure traversal routine is given below:

/* Render model by recursively traversing the tree (or DAG) structure. */
void render_object(struct object *curr_object)
{
    if (curr_object != NULL) {
        /* Push stack so we can come back. */
        glPushMatrix();

        /* Transformations for this object. */
        glTranslatef(...);
        glScale(...);
        glRotate(...);

        /* Draw object. */
        switch (curr_object->type) {
            case box:
                draw_box( .....);
                break;
            /* Add other objects here */
            default:
                printf(‘Error - unknown object type\n’);
                break;
        }

        /* Render next object lower in the hierarchy. */
        render_object(curr_object->next_level);

        /* Restore transformation matrix. */
        glMatrixMode();

        /* Render next object at same level in the hierarchy. */
    }
render_object(curr_object->next_object);
}
}

(d)

Projectile motion

In the vertical direction

\[ v = v_0 - gt \]
\[ y = v_0 t - \frac{1}{2}gt^2 \]

The ball is caught at the same height it is thrown. Therefore it must be at its maximum height at half the total time it is in the air, i.e., \( t = 1 \) s. The ball will also be at its minimum vertical velocity of 0 m/s at \( t = 1 \).

\[ v = v_0 - gt \]
\[ v_0 = v + gt \]
\[ = 0 + (10.0 \text{ m/s}^2)(1.0 \text{ s}) \]
\[ = 10.0 \text{ m/s} \]

The initial velocity of the ball is 10 m/s.

\[ y = v_0 t - \frac{1}{2}gt^2 \]
\[ = (10 \text{ m/s})(1 \text{ s}) - \frac{1}{2}(10 \text{ m/s}^2)(1 \text{ s})^2 \]
\[ = 5 \text{ m} \]

The maximum height of the ball is 5 m.

To find the horizontal velocity we need to know the horizontal distance travelled. From the diagram we can see the distance is -13 m. The horizontal velocity is

\[ v = \frac{\Delta x}{\Delta t} \]
\[ = -13 \text{ m}/2 \text{ s} \]
\[ = -6.5 \text{ m/s} \]

(e)

The diagrams for the juggler’s right shoulder, elbow and wrist are shown below