(a) In each the diagrams below four 3D coordinate systems are drawn with one axis missing. The two axes shown lie in the plane of the page. The coordinate systems are left or right handed as labelled. Draw each coordinate system in your exam booklet including the missing axis and state whether the missing axis points into or out of the plane of the page.

(b) Triangle $T_1$ has the vertices $A(1,1,2)$, $B(5,0,-5)$ and $C(3,5,-6)$. Work out the plane equation of the plane in which $T_1$ lies.

(c) Triangle $T_2$ has the vertices $D(2,2,-5)$, $E(3,3,-8)$ and $F(4,1,-8)$. Work out the plane equation of the plane in which $T_2$ lies.

(d) Draw a diagram showing the (orthographic) parallel projection of $T_1$ and $T_2$ onto the $z = 0$ or $xy$ plane along the $z$-axis.

(e) Assuming the projection in (d) and the use of a z-buffer to solve the hidden surface problem, which polygon would be visible at $z = 3, y = 2$.

(f) Again assuming the projection in (d) but this time no z-buffer, show the result of drawing: (i) $T_1$ first and $T_2$ second and (ii) $T_2$ first and $T_1$ second. Justify your answer.

\[(1+4+4+1+2+2+3 = 15 \text{ marks})\]
Question 2

(a) In OpenGL 4 x 4 transformation matrices are used for 3D graphics rather than 3 x 3 transformation matrices. Explain why.

(b) Give the transformation steps and the combined transformation matrix (CTM) which transforms the object shown below on the left into the object shown below on the right.

(c) Assume a function void displayObject(void) which draws the object as shown on the left in the diagram in (b) above is available. Write an OpenGL code fragment which includes the transformations needed to draw the object on the right.

(4+7+3 = 14 marks)

Question 3

(a) The scan-line algorithm for rasterizing polygons uses an edge table and an active edge table. Give the algorithm, explain what the edge table and active edge table are and explain how they are used to achieve an efficient algorithm. Include in your explanation a discussion of preprocessing, coherence and incremental arithmetic.

(b) For the polygon ABCDEFGHI below draw a diagram showing (i) the initial edge table and (ii) the active edge table at the start of scan lines y = 4, y = 5, y = 7 and y = 11 Indicate the pixels which will be intensified on each of those scan lines.

(c) Draw diagrams of convex, concave and non-simple or bowtie polygons and indicate whether the polygon rasterization algorithm you gave in part (a) will handle each type of polygon. Discuss what changes to the algorithm you gave can be made for each type of polygon, using diagrams as appropriate.

(6+6+3 = 15 marks)
Shown below is a simple humanoid model consisting of a torso, head and arms. The arms have clavicle, upper arm, lower arm and hand segments, and sternoclavicular, shoulder, elbow and wrist joints. Each arm segment is drawn as a box and each arm joint is drawn as a sphere. The joints are capable of being rotated around the $z$ axis only. The model is shown with all joint angles at zero.

(a) Assume there is a function `void wireUnitSphere(void)` which draws an outline of a sphere of diameter one unit centred at the origin. Using OpenGL routines, write a function `void myWireSphere(float d)` which uses `wireUnitSphere()` to draw a scaled sphere of diameter $d$.

(b) Assume there is a function `void wireUnitCube(void)` which draws an outline of a cube of width, height and depth one unit centred at the origin. Using OpenGL routines, write a function `void myWireBox(float w, float h, float d)` which uses `wireUnitCube()` to draw a scaled cube, i.e., a box, of width $w$, height $h$ and depth $d$.

(c) Write a function `void display(void)` which uses OpenGL modelling transformations, matrix stack manipulation, `myWireSphere(d)` and `myWireBox(w, h, d)` to draw the model. The rotation angles are to be specified by global variables corresponding to the name of the joints, that is, `leftSternoClavicular`, `leftShoulder`, `leftElbow` and `leftWrist` and likewise for the right arm. Assume all viewing parameters, window initialisation, etc., have already been correctly set up.

(d) In the diagram below key frames of a wave animation of the humanoid’s left arm are shown. Assuming constant angular velocity, give parameter curves for the humanoid’s joints for this animation.

(e) Give the values of the left arm joints at $t = 2$ and $t = 4$.

(f) Write a function `void interpolate(float elapsedTime)` which uses linear interpolation to work out the left arm joint angles and store them in the corresponding global variables. Assume the argument `elapsedTime` is the number of seconds since the animation started. Use an approach of your choice to storing the key frame values with a brief explanation.

$$2+3+10+5+3+7 = 30\text{ marks}$$
Question 5

(a) Give the four cases or relationships between a polygon edge and a clip edge, and the output in each case, as used in the Sutherland-Hodgeman polygon clipping algorithm.

(b) Show the steps the Sutherland-Hodgeman clipping algorithm would take when clipping the polygon ABCDEFGHI against the clip boundary JKLM.

(c) To achieve efficiency the Cohen-Sutherland line clipping algorithm performs trivial rejection and trivial acceptance tests and only if they fail are more expensive calculations used. Explain how the Cohen-Sutherland line clipping algorithm performs the trivial rejection and trivial acceptance tests.

\[4 + 6 + 4 = 14 \text{ marks}\]

Question 6

In the diagram below a (truncated) cone has been rendered in five different ways using OpenGL.

(a) Give OpenGL settings or modes which may produce each of the images, including the values which may be given to some or all of the following OpenGL routines: glColor, gIPolygonMode, glEnable, glShadeModel, glMaterial, gILightModeli, glColorMaterial.

(b) Give and explain a diagram showing the result of calling glEnable(GL_CULL_FACE) for the first rendering.

(c) The OpenGL lighting model includes emissive, ambient, diffuse and specular components. Explain what you understand of each of these and how they are combined. Use diagrams and mathematical formulae in your answer as appropriate.

\[5 + 2 + 5 = 12 \text{ marks}\]

THE END