Results of the 2013 IEEE CEC Competition on Niching Methods for Multimodal Optimization

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Outline

1. Introduction
2. Participants
3. Results
4. Winners
5. Summary
Numerical optimization is probably one of the most important disciplines in optimization. Many real-world problems are “multimodal” by nature, i.e., multiple satisfactory solutions exist.

**Niching methods:** promote and maintain formation of multiple stable subpopulations within a single population.

- **Aim:** maintain diversity and locate multiple globally optimal solutions.

**Challenge:** Find an efficient optimization algorithm, which is able to locate multiple global optimal solutions for multimodal problems with various characteristics.
Provide a common platform that encourages fair and easy comparisons across different niching algorithms.


- 20 benchmark multimodal functions with different characteristics
- 5 accuracy levels: $\varepsilon \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$
- The benchmark suite and the performance measures have been implemented in: C/C++, Java, MATLAB
**Benchmark function set**


<table>
<thead>
<tr>
<th>Id</th>
<th>Dim.</th>
<th># GO</th>
<th>Name</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$</td>
<td>1</td>
<td>2</td>
<td>Five-Uneven-Peak Trap</td>
<td>Simple, deceptive</td>
</tr>
<tr>
<td>$F_2$</td>
<td>1</td>
<td>5</td>
<td>Equal Maxima</td>
<td>Simple</td>
</tr>
<tr>
<td>$F_3$</td>
<td>1</td>
<td>1</td>
<td>Uneven Decreasing Maxima</td>
<td>Simple</td>
</tr>
<tr>
<td>$F_4$</td>
<td>2</td>
<td>4</td>
<td>Himmelblau</td>
<td>Simple, non-scalable, non-symmetric</td>
</tr>
<tr>
<td>$F_5$</td>
<td>2</td>
<td>2</td>
<td>Six-Hump Camel Back</td>
<td>Simple, not-scalable, non-symmetric</td>
</tr>
<tr>
<td>$F_6$</td>
<td>2,3</td>
<td>18,81</td>
<td>Shubert</td>
<td>Scalable, #optima increase with D, unevenly distributed grouped optima</td>
</tr>
<tr>
<td>$F_7$</td>
<td>2,3</td>
<td>36,216</td>
<td>Vincent</td>
<td>Scalable, #optima increase with D, unevenly distributed optima</td>
</tr>
<tr>
<td>$F_8$</td>
<td>2</td>
<td>12</td>
<td>Modified Rastrigin</td>
<td>Scalable, #optima independent from D, symmetric</td>
</tr>
<tr>
<td>$F_9$</td>
<td>2</td>
<td>6</td>
<td>Composition Function 1</td>
<td>Scalable, separable, non-symmetric</td>
</tr>
<tr>
<td>$F_{10}$</td>
<td>2</td>
<td>8</td>
<td>Composition Function 2</td>
<td>Scalable, separable, non-symmetric</td>
</tr>
<tr>
<td>$F_{11}$</td>
<td>2,3,5,10</td>
<td>6</td>
<td>Composition Function 3</td>
<td>Scalable, non-separable, non-symmetric</td>
</tr>
<tr>
<td>$F_{12}$</td>
<td>2,3,5,10</td>
<td>8</td>
<td>Composition Function 4</td>
<td>Scalable, non-separable, non-symmetric</td>
</tr>
</tbody>
</table>
Measures:

**Peak Ratio** (PR) measures the average percentage of all known global optima found over multiple runs:

$$PR = \frac{\sum_{run=1}^{NR} \# \text{ of Global Optima}_i}{(\# \text{ of known Global Optima}) \times (\# \text{ of runs})}$$

Who is the winner:

- The participant with the highest average Peak Ratio performance on all benchmarks wins.
- In all functions the following holds: the higher the PR value, the better.
Participants

Submissions to the competition:

- E-1682: **(PNA-NSGAII)** A Parameterless-Niching-Assisted Bi-objective Approach to Multimodal Optimization
- E-1419: **(N-VMO)** Variable Mesh Optimization for the 2013 CEC Special Session Niching Methods for Multimodal Optimization
- E-1449: **(dADE/nrand/1,2)** A Dynamic Archive Niching Differential Evolution algorithm for Multimodal Optimization
- Mike Preuss: **(NEA1, NEA2)** Niching the CMA-ES via Nearest-Better Clustering [2]
Participants (2)

**Implemented algorithms for comparisons:**

- **(A-NSGAII)** A Bi-objective NSGA-II for multimodal optimization (taken from E-1682)[1]
- **(CrowdingDE)** Crowding Differential Evolution [3]
- **(DE/nrand/1,2)** Niching Differential Evolution algorithms with neighborhood mutation strategies [5]
- **(CMA-ES, IPOP-CMA-ES)** CMA-ES/IPOP-CMA-ES with a restart procedure and a dummy archive. [6,7]

Mike Preuss: CMA-ES, IPOP-CMA-ES, MG Epitropakis: DE/nrand/1,2, DECG, DELG, DELS-aj, CrowdingDE
Results

Summary:

- 4 submissions/teams from six countries (four continents)
- 15 algorithms
- 20 benchmark functions
- 5 accuracy levels $\varepsilon \in \{10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}\}$

Results: per accuracy level & over all accuracy levels
Accuracy level $\varepsilon = 10^{-1}$

Accuracy level 1.0e−1

Benchmark function

A-NSGAII
CMA-ES
CrowdingDE
dADE/nrand/1
dADE/nrand/2
DECG
DELS~aj
DE/nrand/1
DE/nrand/2
IPOP-CMA-ES
NEA1
NEA2
N-VMO
PNA-NSGAII

Peak Ratio in all benchmark functions
Results

Accuracy level $\epsilon = 10^{-2}$

Accuracy level $1.0e^{-2}$

Benchmark function
5
10
15
20
A−NSGAII
CMA−ES
CrowdingDE
dADE/nrand/1
dADE/nrand/2
DECG
DELG
DELS−aj
DE/nrand/1
DE/nrand/2
IPOP−CMA−ES
NEA1
NEA2
N−VMO
PNA−NSGAII

Peak Ratio in all benchmark functions

Algorithms
- A−NSGAII
- CMA−ES
- CrowdingDE
dADE/nrand/1
dADE/nrand/2
DECG
DELG
DELS−aj
DE/nrand/1
DE/nrand/2
IPOP−CMA−ES
NEA1
NEA2
N−VMO
PNA−NSGAII

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Results

Accuracy level $\varepsilon = 10^{-3}$

Accuracy level $1.0\times10^{-3}$

Benchmark function

A-NSGAII
CMA-ES
CrowdingDE
dADE/nrand/1
dADE/nrand/2
DECG
DELG
DELS-aj
DE/nrand/1
DE/nrand/2
IPOP-CMA-ES
NEA1
NEA2
N-VMO
PNA-NSGAII

Peak Ratio in all benchmark functions

Algorithms

Accuracy level 1.0e−3
Accuracy level $\varepsilon = 10^{-4}$
Accuracy level $\epsilon = 10^{-5}$

![Graph showing results of different algorithms for accuracy level $1.0 \times 10^{-5}$]

**Algorithms**
- A-NSGAII
- CMA-ES
- CrowdingDE
- dADE/nrand/1
- dADE/nrand/2
- DECG
- DELG
- DELS-aj
- DE/nrand/1
- DE/nrand/2
- IPOP-CMA-ES
- NEA1
- NEA2
- N-VMO
- PNA-NSGAII

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Overall performance (1)

Results

Peak Ratio in all benchmark functions

All Accuracy levels

Algorithms

A–NSGAII
CMA–ES
CrowdingDE
dADE/nrand/1
dADE/nrand/2
DECG
DELG
DELS–aj
DE/nrand/1
DE/nrand/2
IPOP–CMA–ES
NEA1
NEA2
N–VMO
PNA–NSGAII

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## Overall performance (2)

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Median</th>
<th>Mean</th>
<th>St.D.</th>
<th>Rank</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-NSGAII</td>
<td>0.0740</td>
<td>0.3275</td>
<td>0.4044</td>
<td>15</td>
<td>3.1450</td>
</tr>
<tr>
<td>CMA-ES</td>
<td><strong>0.7550</strong></td>
<td><strong>0.7137</strong></td>
<td>0.2807</td>
<td>3</td>
<td><strong>10.2300</strong></td>
</tr>
<tr>
<td>CrowdingDE</td>
<td>0.6667</td>
<td>0.5731</td>
<td>0.3612</td>
<td>8</td>
<td>7.7900</td>
</tr>
<tr>
<td>dADE/nrand/1</td>
<td><strong>0.7488</strong></td>
<td><strong>0.7383</strong></td>
<td>0.3010</td>
<td>2</td>
<td><strong>10.6700</strong></td>
</tr>
<tr>
<td>dADE/nrand/2</td>
<td>0.7150</td>
<td>0.6931</td>
<td>0.3174</td>
<td>5</td>
<td>9.6200</td>
</tr>
<tr>
<td>DECG</td>
<td>0.6567</td>
<td>0.5516</td>
<td>0.3992</td>
<td>13</td>
<td>6.4950</td>
</tr>
<tr>
<td>DELG</td>
<td>0.6667</td>
<td>0.5706</td>
<td>0.3925</td>
<td>11</td>
<td>7.0350</td>
</tr>
<tr>
<td>DELS-aj</td>
<td>0.6667</td>
<td>0.5760</td>
<td>0.3857</td>
<td>12</td>
<td>7.0250</td>
</tr>
<tr>
<td>DE/nrand/1</td>
<td>0.6396</td>
<td>0.5809</td>
<td>0.3338</td>
<td>9</td>
<td>7.7600</td>
</tr>
<tr>
<td>DE/nrand/2</td>
<td>0.6667</td>
<td>0.6082</td>
<td>0.3130</td>
<td>6</td>
<td>8.3200</td>
</tr>
<tr>
<td>IPOP-CMA-ES</td>
<td>0.2600</td>
<td>0.3625</td>
<td>0.3117</td>
<td>14</td>
<td>3.8900</td>
</tr>
<tr>
<td>NEA1</td>
<td>0.6496</td>
<td>0.6117</td>
<td>0.3280</td>
<td>10</td>
<td>7.6300</td>
</tr>
<tr>
<td>NEA2</td>
<td><strong>0.8513</strong></td>
<td><strong>0.7940</strong></td>
<td>0.2332</td>
<td>1</td>
<td><strong>11.9300</strong></td>
</tr>
<tr>
<td>N-VMO</td>
<td><strong>0.7140</strong></td>
<td><strong>0.6983</strong></td>
<td>0.3307</td>
<td>4</td>
<td><strong>10.1550</strong></td>
</tr>
<tr>
<td>PNA-NSGAII</td>
<td>0.6660</td>
<td>0.6141</td>
<td>0.3421</td>
<td>7</td>
<td>8.3050</td>
</tr>
</tbody>
</table>
Winners

Ranking based on average PR values

1. **NEA2** (Mike Preuss) Niching the CMA-ES via Nearest-Better Clustering
2. **dADE/nrand/1** (E-1449) A Dynamic Archive Niching Differential Evolution algorithm
3. **CMA-ES** (Mike Preuss) CMA-ES with simple archive
4. **N-VMO** (E-1419) Niching Variable Mesh Optimization algorithm

Note: The algorithms have not been fine-tuned for the specific benchmark suite!
Conclusions

Summary

- Four teams from six countries (four continents)
- **Winner:** NEA2 (Mike Preuss) Niching the CMA-ES via Nearest-Better Clustering
  - Competitive on average performance, (nearest-better clustering, archive mechanism, CMA-ES)
- Places 2 to 4 very close:
  - dADE/nrand/1 (E-1449) A Dynamic Archive Niching Differential Evolution algorithm
  - CMA-ES (Mike Preuss) CMA-ES with simple archive
  - N-VMO (E-1419) Niching Variable Mesh Optimization algorithm

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Conclusions (2)

- The competition gave a boost to the multimodal optimization community
- New competitive and very promising approaches

Key characteristics of the algorithms:

- Many attempts to overcome the influence of the algorithm’s parameters (niching parameters, population size)
- Usage of Archives to maintain good solutions
- Multiobjectivization, Clearing, Clustering and neighborhood mutation-based niching techniques
- Algorithms: Differential Evolution, CMA-ES, Variable Mesh Optimization and NSGAII
Future Work

Possible objectives:

- Re-organize the competitions in future
- Enhance the benchmark function set
- Introduce new performance measures
- Automate the experimental design and results output
- Boost multimodal optimization community
We really want to thank for their help:

- The participants :-) 
- Dr. Jerry Swan, University of Stirling, Scotland, UK 
- Dr. Mike Preuss, TU Dortmund, Germany 
- Dr. Daniel Molina Cabrera, University of Cadiz, Spain 
- Dr. Catalin Stoean, University of Craiova, Romania
Thank you very much for your attention :-)

Questions ???

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