Robust Congestion Control for High Speed Data Networks with Uncertain Time-Variant Delays: an LMI Control Approach

J. Hu *, *Member IEEE,* J. Lin and L. Xie ∓, *Senior Member IEEE* ∓

* *School of Computer Science and IT, RMIT University, Melbourne 3000, Australia.*
Email: Jiankun@cs.rmit.edu.au

∓ *School of Electrical and Electronic Engineering, BLK S2, Nanyang Technological University, Singapore 639798, Email: elhxie@ntu.edu.sg*

Abstract

In this paper, we first develop a delay-dependent condition for the stability and H_∞ performance of systems with time-variant delays in both the state and output equations in terms of an LMI (Linear Matrix Inequality). The analysis result is then applied to derive a H_∞ congestion control where the congestion problem is formulated as the H_∞ control of systems with time-variant input delays. Illustrative examples are provided to show excellent performance of the proposed algorithm in achieving an equilibrium in the buffer occupancy in the presence of time-variant delays. To the best of our knowledge, no such congestion control approach that directly accommodates the issue of the uncertain time-variant delays has been reported.

1. Introduction

Recently a macroscopic model has been proposed to address time-varying delay congestion problem in ATM networks [2]. Though it is not clearly indicated, this conclusion depends on the HFS and VBR models where several assumptions have been made. One assumption is that a source will use the same sending rate as before if no fresher sample has arrived. However, there exist many other ways to handle unavailable cells such as the worst-case estimate of the feedback information [4]. Therefore, such conclusion should be restricted to where HFS and VBR models are used. In this paper, we propose a more practical solution based on LMI (linear matrix inequality) theory [3][4] to the controller design problem for time variant delay systems. The mathematical model of the problem description and assumptions are adopted from [1][2] with the following modifications: (i) the time invariant delays in [1][2] are now time-variant; (ii) the zero-mean i.i.d noise introduced in the AR model for the description of stochastic available bandwidth is now replaced with an energy bounded noise. This modification covers wider modelling errors which are more realistic.

The proposed control scheme can ensure robust performance in the presence of time variant delays and does not require us to solve an extended state space problem suffered by standard linear-quadratic optimal control. Illustrative simulation examples have shown an excellent performance of the proposed algorithm in achieving an equilibrium in the buffer occupancy in the presence of time-variant delays.

2. System description

Let q_k be the queue length at the bottleneck and μ_k the effective service rate available for the traffic of the given source in that link at the beginning of the *k*th time slot. Let r_k denote the effective source rate measured at the congestion switch. Without loss of generality, we consider the case of single connection. Therefore, the queue length equation is given by

$$q_{k+1} = q_k + r_k - \mu_k . \quad (1)$$

The ARMA disturbance model is given as

$$\mu_k = \mu + \zeta_k$$
$$\zeta_{k+1} = \sum_{i=1}^{p} l_i \zeta_{k+1-i} + w_k \quad (2)$$

where μ is the constant nominal service rate, and l_k are known parameters. $\{w_k\}_{k\geq 1}$ is assumed to be an energy bounded signal. Note that no statistics of w_k has been assumed. Assume that there is no cell loss.

0-7695-2421-4/05 $20.00 © 2005 IEEE

The delay between u_k and r_k is d_k which is the round trip delay. In a wide area network, propagation delay dominates and hence a constant delay is used. In order to take into account the jitter of round trip time due to queuing time, a model containing time varying delays should be considered [4]. Therefore, we have the following relationship,

$$r_k = u_{k-d_k} \,. \qquad (3)$$

In this paper, we formulate the congestion control problem as an H_∞ control problem whose objective is to achieve both the system stability and the performance index

$$J_\infty = \sup_{w\neq 0} \frac{\sum_{k=1}^{\infty}\left[(q_k - q_d)^2 + e^2 (r_k - \mu_k)^2\right]}{\sum_{k=1}^{\infty} w_k^2} < \gamma^2 \qquad (4)$$

for some pre-specified $\gamma > 0$, where q_d is the desired queue length, e is a constant. The first term of the numerator represents a penalty for deviating from a desired queue length. The second term is a measure of the quality with which the input rate tracks the available link capacity, and e is the weighting to balance the importance of these two terms.

3. Controller design

Introduce

$$x(k) = [(q_k - q_d), \xi_{k+1-p}, ..., \xi_k]^T \,, \qquad (5)$$

$v_k = u_k - \mu$, $z_1(k) = q_k - q_d$, $z_2(k) = r_k - \mu_k = v_{k-d_k} - \xi_k$, where $0 < d_k \le m$ with known upper bound m. Based on the system description given in Eq.(5), set the feedback controller $v(k) = Fx(k)$, then it has the following H_∞ performance problem for the system

$$\begin{aligned} x(k+1) &= Ax(k) + A_1 x(k - d_k) + B_1 w_k \\ z(k) &= C_1 x(k) + D_1 x(k - d_k) + E_1 w_k \end{aligned} \qquad (6)$$

where $w \in \ell_2[0,\infty)$ and $1 \le d_k \le m$. Given a scalar $\gamma > 0$, the system is said to have the H_∞ performance γ if the system is robustly stable and under zero initial condition ($x(k) = 0, -d_0 \le k \le 0$).

Theorem 1. Given a scalar $\gamma > 0$, the system (6) has the H_∞ performance γ if there exist matrices P, Q, S, R and Z such that the following matrix inequalities hold:

$$\Gamma = \begin{bmatrix} M & -R & 0 & A^T P & m(A-I)^T Z & C_1^T \\ * & -Q & 0 & A_1^T P & mA_1^T Z & D_1^T \\ * & * & -\gamma^2 I & B_1^T P & mB_1^T Z & E_1^T \\ * & * & * & -P & 0 & 0 \\ * & * & * & * & -mZ & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0$$

(7a)

$$\begin{bmatrix} S & R \\ R^T & Z \end{bmatrix} \ge 0 \qquad (7b)$$

where

$$M = -P + mS + R + R^T + mQ \,. \qquad (8)$$

Proof: omitted due to limited space.

4. Simulation

Two simulation examples have been conducted. One example has one source and another example has multiple sources. In these two cases, both feedback and forward path have random time-varying delays. The robust controllers have been obtained via the proposed Theorem 1. As shown in the simulation results, the buffer level clearly keeps tracking the equilibrium with small fluctuations to compensate for the random fluctuations in the bandwidth available. The result has demonstrated a very good robust performance for our design. Detailed results are unable to be provided here due to tight space.

Acknowledgement: The work is supported by ARC Linkage **LP0455234**

5. References

[1] E. Altman, T. Basar and R. Srikant, "Congestion control as a stochastic control problem with action delays", *Automatica*, vol. 35, 1999, pp. 1937-1950.

[2] M.L. Sichitiu, P.H. Bauer and K. Premaratne, "The effect of uncertain time-variant delays in ATM networks with explicit rate feedback: a control theoretic approach," *IEEE/ACM Transactions on Networking*, vol. 11, no.4, 2003, pp.628-637.

[3] H. Gao, J. Lam, C. Wang and Y. Wang, "Delay-dependent output-feedback stabilisation of discrete-time systems with time-varying state delay", *IEE Proc. Control Theory Appl.*, vol. 151, no.6, 2004, pp 691-698.

[4] Further references are omitted due to limited space.