Topic 5
Kinematics

References

Notes prepared from

Halliday, D., Resnick, R. and Walker, J., Fundamentals of Physics, Fifth Edition, Wiley, 1997. Chapter 2.

Kinematics

Mechanics is the study of forces and motion. Mechanics may be broken into two subdomains: *statics* and *dynamics*.

Statics is the study of forces and motion where there is no motion.

Dynamics is the study of forces and motion where bodies are moving.

Dynamics can be further broken into two subfields: *kinematics* and *kinetics*.

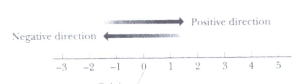
Kinematics is the study of motion without taking into account how motion is produced, that is, without taking into account forces.

Kinetics takes forces, and energy, into account.

Mechanics	Force	Motion
Statics	X	.
Dynamics	X	X
Kinetics	X	X
Kinematics	.	X

Motion Along a Straight Line

We first consider motion along a straight line: the x-axis.



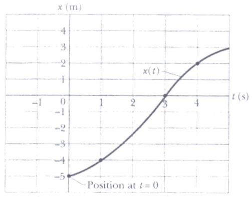
The *position* of an object is its distance from a reference point. The reference point is commonly called the origin. The line may be called the x-axis.

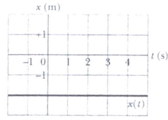
The *displacement* Δx of an object which moves from x_1 to x_2 is

$$\Delta x = x_2 - x_1 \quad (8)$$

Displacement is a *vector* quantity. It has a direction and a magnitude.

A compact way to describe position is with a *position curve* — a graph of position x plotted against time t.



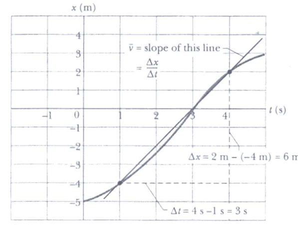


One measure of how fast an object moves is *average velocity.*

The average velocity of an object is the ratio of a displacement Δx to the time interval Δt in which the displacement occurs. It is a vector quantity.

$$\overline{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \tag{9}$$

The average velocity $\overline{v}$ is the *slope* of the straight line between the points (x_1, t_1) and (x_2, t_2) on a position curve.



Another measure of how fast an object moves is its *average speed.* Average speed is *total distance* divided by change in time rather than *displacement* divided by change in time.

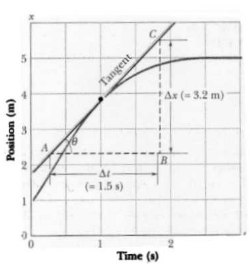
$$\overline{s} = \frac{totaldistance}{\Delta t} \tag{10}$$

Average speed is a *scalar* value rather than a *vector* value.

The *instantaneous velocity* or simply *velocity* of an object is how fast an object is moving at a particular point in time.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{11}$$

The instantaneous velocity v is the slope of tangent to the position curve at the point representing that instant.



The *instantaneous speed* of an object is the magnitude of its instantaneous velocity v. It is a scalar.

An object whose velocity changes is said to undergo *acceleration.*

The *average acceleration* $\overline{a}$ of an object over an interval of time Δt is

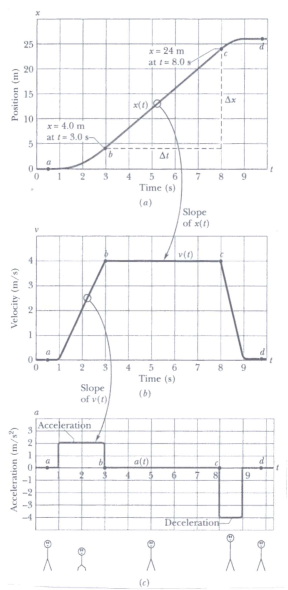
$$\overline{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \tag{12}$$

The *instantaneous acceleration* or simply *acceleration* a of an object over an interval of time Δt is

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \tag{13}$$

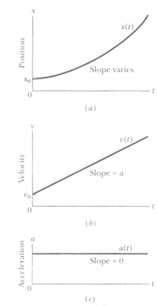
Motion Curves

To describe the motion of an object we can plot x, v and a as a function of t.



Constant Acceleration

A common type of motion is where constant acceleration occurs.



In the case of constant acceleration, average acceleration and instantaneous acceleration have the same value:

$$a = \frac{v - v_0}{t - 0} \qquad (14)$$

where at $t = 0$, $v = v_0$.

Rearranging we get

$$v = v_0 + at \qquad (15)$$

Likewise we can rewrite 9 to give

$$x = x_0 + \overline{v}t \qquad (16)$$

A plot of v against t gives a straight line. The average velocity for an interval Δt from $t = 0, v = v_0$ to t, v is

$$\overline{v} = 1/2(v_0 + v) \qquad (17)$$

Substituting for v from equation 15 gives

$$\overline{v} = v_0 + 1/2at \qquad (18)$$

Substituting 18 into 16 gives

$$x - x_0 = v_0t + 1/2at^2 \qquad (19)$$

Five quantities — $x - x_0, v, t, a$ and v_0 — are involved in a constant acceleration problem. The

equations above can be substituted into each other in various ways to eliminate one of them.

The following table summarises the results:

$$\begin{aligned}
v &= v_0 + at && (20)\\
x - x_0 &= v_0t + 1/2at^2 && (21)\\
v^2 &= v_0^2 + 2a(x - x_0) && (22)\\
x - x_0 &= 1/2(v_0 + v)t && (23)\\
x - x_0 &= vt - 1/2at^2 && (24)
\end{aligned}$$

Free Fall (Gravity)

When objects are in free fall, that is, moving vertically up or down with only gravity acting on them, they undergo constant acceleration.

The value of the acceleration is g which at the earth's surface is approximately $9.8m/s^2$.

The direction of the acceleration is *down*.

The equations for constant acceleration then become

$$v = v_0 - gt \quad (25)$$

$$y - y_0 = v_0 t - 1/2gt^2 \quad (26)$$

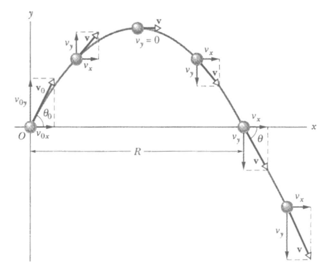
$$v^2 = v_0^2 - 2g(y - y_0) \quad (27)$$

$$y - y_0 = 1/2(v_0 + v)t \quad (28)$$

$$y - y_0 = vt + 1/2gt^2 \quad (29)$$

Projectile Motion

An object which is thrown, shot, or which jumps, from one position to another undergoes what is termed *projectile motion*.



In projectile motion an object is in free fall in the vertical direction. That is it undergoes constant acceleration — with the value of gravity — in the vertical direction. It has constant velocity in the horizontal direction.

Thus the motion of a projectile, such as a juggling ball, may be separated into two components: vertical and horizontal.

The vertical component follows the laws of motion for constant acceleration, in particular, free fall.

The horizontal component follows the laws of motion for constant velocity.

Thus if the initial velocity v_0 is given as a vector $v_0 = (v_{0x}, v_{0y})$ then the object's vertical position follows the path

$$y - y_0 = v_{0y} t - 1/2gt^2 \quad (30)$$

The horizontal position follows the path

$$x - x_0 = v_{0x} t \quad (31)$$

If instead of vector the initial velocity is given by an angle and a magnitude then the equations become

$$y - y_0 = (v_0 sin\theta_0)t - 1/2gt^2 \quad (32)$$

The horizontal position follows the path

$$x - x_0 = v_0 cos\theta_0 t \quad (33)$$