Instructions to Candidates:

The exam accounts for 50% of the total marks for the subject.

Candidates should attempt all questions 1 to 7.

Marks for each question are shown. Total marks for the exam is 120.

Question 1

Give a concise answer to each of the following questions:

(a) Draw triangles which allow the values of $\cos(30^\circ)$, $\sin(30^\circ)$ and $\cos(45^\circ)$ to be determined.

(b) Normalise the vector $(1, -1, 1)$.

(c) What does a dot product value of 1 between two normalised vectors signify?

(d) What does framerate mean?

(e) How do rasterization and scan-conversion differ?

(f) What mathematical property ensures that the matrix products $(AB)C$ and $A(BC)$ are the same?

(g) What is a GPU?

(h) What 3D points do the homogeneous coordinates $(2, 4, -6, 2)$ and $(4, 8, -12, 4)$ represent?

(i) How does a programmer request a depth buffer using $\texttt{glut}$?

(j) Name two OpenGL matrix stacks.

(1+1+1+1+1+1+1+1+1 = 10 marks)

Question 2

(a) Given the two vectors:

$\vec{A} = 3i - 2j + k$

$\vec{B} = i + 2j - k$

Calculate or perform each of the following, showing your working:

(i) $\vec{A} - \vec{B}$

(ii) $|\vec{B}|$

(iii) a unit vector in the direction of $\vec{B}$

(iv) $\vec{A}.\vec{B}$

(v) $\vec{A} \times \vec{B}$
(vi) the cosine of the angle between $\vec{A}$ and $\vec{B}$

(b) Triangle $T_1$ has vertices $A(2, 5, 2), B(4, 1, 1)$ and $C(5, 7, 0)$. Work out the plane equation of the plane in which $T_1$ lies.

(c) A second triangle $T_2$ has the vertices $D(2, 3, 2), E(6, 2, 0)$ and $F(7, 6, 1)$. If the hidden surface problem was to be solved by drawing the polygons from back to front, what order should they be drawn in for a viewer who is positioned at $(4,4,10)$ looking in the direction of $(4,4,0)$. Show your working.

$$((1+1+1+2+2+2)+4+5 = 18 \text{ marks})$$

**Question 3**

(a) Transformations are implemented in OpenGL using $4 \times 4$ matrices. Explain why $3 \times 3$ matrices are not used, given that OpenGL is a 3D graphics library.

(b) An axe, as shown in the diagram on the left below, is being used as a weapon in a game. Give the transformation steps and the combined transformation matrix (CTM) to transform the axe into the situation shown on the right.

![Diagram of axe and its transformation]

(c) Write a code fragment which shows the OpenGL function calls necessary to transform the axe on the left to the one shown on the right in the diagram in (b) above. Assume there is a function `void displayAxe(void)` which draws the axe as shown on the left.

$$((4+10+3 = 17 \text{ marks})$$

**Question 4**

(a) Bresenham’s line rasterization algorithm is given below:

```c
/* value is colour value of line to be drawn */
void MidpointLine(int x0, int y0, int x1, int y1, int value)
{
    int dx, dy, incrE, incrNE, d, x, y;
    dx = x1 - x0;
    dy = y1 - y0;
    d = 2 * dy - dx; /* initialise decision variable */
    incrE = 2 * dy; /* increment for E direction */
    incrNE = 2 * (dy - dx); /* increment for NE direction */
    x = x0;
    y = y0;
    WritePixel(x, y, value); /* Fill in the initial pixel */
    while (x < x1)
    {
        if (d < 0) /* select the E direction */
            d += incrE;
        else /* select the NE direction */
            d += incrNE;
        x += 1;
        y += dy; /* calculate next pixel */
    }
}
```


else { /* else, select the NE direction */
    d += incrNE;
    y++;
} 
    x++;
    WritePixel(x, y, value);
}

Show in detail the steps performed by Bresenham’s line rasterization algorithm for the line (0,1) - (8,4):

(b) Explain the purpose of the variable d in Bresenham’s line rasterization algorithm.

(c) Give a polygon rasterization algorithm which operates in a scan-line by scan-line manner and uses an edge table and an active edge table.

(d) Explain the purpose of the edge table and active edge table and how they are used to improve performance.

(e) What is incremental arithmetic? Does the algorithm you gave as answer to (c) use incremental arithmetic? If so explain how.

(f) For the polygon below draw a diagram showing (i) the initial edge table and (ii) the active edge table at the start of scan lines y = 2, y = 3, y = 5, y = 6 and y = 7. Indicate the pixels which will be intensified on each of those scan lines.

(g) Will the pixel (8,16) be drawn? Why or why not?

(h) OpenGL only handles convex polygons. Some applications need to use concave polygons. Explain what convex and concave polygons are and how an application using concave polygons can still use OpenGL.

(5+2+3+4+2+4+2+3 = 25 marks)
Question 5

(a) The Sutherland-Hodgeman polygon clipping algorithm uses four cases or relationships between a polygon edge and a clip edge to determine what should be output for that particular polygon edge. Give the four cases and the corresponding output.

(b) Show the steps the Sutherland-Hodgeman clipping algorithm would take when clipping the polygon ABCDEFG against the clip boundary HIJK.

(c) Explain how the Cohen-Sutherland line clipping performs trivial rejection and trivial acceptance tests to help achieve efficiency.

(4+6+4 = 14 marks)

Question 6

Shown below is a simple humanoid model consisting of a torso, head, neck, arms and legs. The model has hip, knee, shoulder, elbow, and at the top and bottom of the neck, skullbase and vc7 joints. Attached to the left arm via a wrist joint is a claw with a left and right plate. The joints are capable of being rotated around the z axis only. The plates of the claw can be translated along the x axis only. The model is shown with all joint angles and plate translations at zero.

(a) Draw a diagram showing the hierarchy of the model.

(b) One of the shapes needed to draw the above figure is a diamond, as used in the legs and arms. Assume there is a function void wireUnitCube(void) which draws an outline of a cube of width one unit centred at the origin. Using OpenGL routines, write a function void myWireDiamond(float l, float h) which uses wireUnitCube() to draw a a diamond of length l and height h.
(c) In the diagram below key frames of an animation of the humanoid’s left arm and claw are shown. Give parameter curves for the humanoid’s arm joints and for the left and right plates of the claw for this animation.

(d) Give the values of the left shoulder and elbow joints at \( t = 2 \) and \( t = 4.5 \).

(e) Write a function \( \text{void displayLeftArmAndClaw(void)} \) which uses OpenGL modelling transformations, matrix stack manipulation, \( \text{myWireDiamond(1,h), wireCube()} \) and a function \( \text{wireSphere()} \) to draw the model’s left arm. The joint rotations and plate translations are to be specified by global variables corresponding to the name of the joints. Assume all viewing parameters, window initialisation, etc., have already been correctly set up.

\( (3+4+5+2+6 = 20 \text{ marks}) \)

**Question 7**

In the diagram below a teapot has been rendered in five different ways using OpenGL.

(a) Give OpenGL settings or modes which may produce each of the images, including the values which may be given to some or all of the following OpenGL routines: \( \text{glColor, glPolygonMode, glEnable, glShadeModel, glMaterial, glLight, glLightModel, glColorMaterial} \).

(b) What does \( \text{glEnable(GL_CULL_FACE)} \) do? How can it be implemented?

(c) Lighting in OpenGL consists of emissive, ambient, diffuse and specular components. Explain each component, using diagrams and mathematical formulae.

(d) OpenGL can be viewed as a state machine. Explain what you understand that means.

\( (5+2+5+4 = 16 \text{ marks}) \)

**THE END**